**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. **0.2676**
4. 0.5
5. 0.6987

**Given:**

**Mean ( *μ* ) = 45 min and Standard deviation (*σ )* = 8 min**

**Sol:**

**Assume X is the amount of time taken to complete the repair on customer’s car. Therefore to finish in one hour: X ≤ 50**

**To find P(X > 50):**

**P(X > 50) = 1 - P(X ≤ 50), Z = (X-*μ*)/*σ* = (X - 45)/8**

**By using the normal table to find:**

**P(X ≤ 50) = P(Z ≤ (50 - 45)/8) = P(Z ≤ 0.625)=73.4%**

**Probability that the service manager will not meet his demand will be**

**= 100-73.4 =26.6% or 0.2676**

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

**Given:**

**Mean ( *μ* ) = 38 and Standard deviation (*σ )* = 6, N=400**

**Sol for A:**

**Probability of employees greater than age 44:**

**P(X>44) =1-P(X ≤ 44), Z = (X -*μ*)/*σ* = (X - 38)/6**

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| **By using the normal table to find:** |
| **P(X ≤ 44) = P(Z ≤ (44 - 38)/6) = P(Z ≤ 1)=84.1345%** |
| **Probability that the employee will be greater than age of 44**  **= 100-84.1345=15.86%** |
| **So the probability of number of employees between 38-44 years of age**  **= P(X<44)-0.5=84.1345-0.5= 34.1345%** |
| **Therefore “More employees at the processing centre are older than 44 than between 38 and 44” is TRUE.** |

**B.**A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**Sol for B: Probability of employees less than age 30:**

**Z = (X - *μ*)/*σ* = (X - 38)/6**

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| **By using the normal table to find:** |
| **P(X ≤ 30) = P(Z ≤ (30 - 38)/6) = P(Z ≤ -1.333)=9.12%** |
| **So the number of employees with the probability 0.912 under age 30=0.912\*400=36.48(36 employees)**    **Therefore statement B is also True** |

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Given: X1 = N (μ1, σ1^2) , X2 =N (μ2, σ2^2) are two independent random variables**

**Sol: Then**

**X1 + X2 = N (μ1 + μ2, σ1^2 + σ2^2)**

**X1 – X2 = N (μ1 − μ2, σ1^2 + σ2^2).**

**Similarly if Z = AX + BY , where X and Y are as defined above, i.e Z is linear combination of X and Y,then Z =N(aμ1 + bμ2, a^2σ1^2 + b^2σ2^2 ).Therefore:**

**2X1~ N (2 u,4 σ^2) and**

**X1+X2 ~ N (μ + μ, σ^2 + σ^2) ~ N (2 u, 2σ^2)**

**2X1-(X1+X2) = N (4μ,6 σ^2)**

1. Let X ~ N(100, 202). Find two values *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. **48.5, 151.5**
6. 90.1, 109.9

**Sol:**

**Have to find the values of a and b, which are symmetric about the mean, such that the probability of random variable taking a value between them is 0.99.Have to work out in reverse order.**

**As we know that the Probability of getting value between a and b should be 0.99.So, the Probability of going wrong or the Probability outside the a and b area is 0.01 (i.e., 1-0.99).**

**The Probability towards left from a = -0.005 (i.e., 0.01/2).**

**The Probability towards right from b = +0.005 (i.e., 0.01/2).**

**So, since we have the probabilities of a and b, we need to calculate X, the random variable at a and b which has got these probabilities.**

**By finding the Standard Normal Variable Z (Z Value), we can calculate the X values: Z= (X- μ) / σ**

**For Probability 0.005 the Z Value is -2.57 (from Z Table).**

**Z \* σ + μ = X**

**Z (-0.005) \*20+100 = -(-2.57) \*20+100 = 151.4**

**Z (+0.005) \*20+100 = (-2.57) \*20+100 = 48.6**

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

**Sol: Range is Rs (99.00810347848784, 980.9918965215122) in Millions**

1. Specify the 5th percentile of profit (in Rupees) for the company

**Sol: 5th percentile of profit (in Million Rupees) is 170.0**

1. Which of the two divisions has a larger probability of making a loss in a given year?

**Sol: Division 1 has a larger probability of making a loss in a given year.**